## Chapter 7

## BOND \& STOCK VALUATION

## 1. OBJECTIVE

\# Use PV to calculate what prices of stocks and bonds should be
! Basic bond terminology and valuation
! Stock and preferred stock valuation
! WSJ price quotation
! market price vs. theoretical price

## \# What is not to be discussed here

! Our mission here is very modest.

Objective: Given CFs that investors are expected to receive and the corresponding required return of a stock, what should the stock's market price be? $\Leftrightarrow$ What is a fair price for the stock?

Note. We are not valuing the company asset or trying to examine what should happen to value of a company if management changes some of its current policies (projects, dividend policy, capital structure)? $\Rightarrow$ objective very modest.
Impact of capital structure on value discussed in a later chapter.

4 In 8 Inis chapter we assume that you are given cash flows and the discount rate " k ". Estimating " k " for bonds will be discussed in the Risk \& Return: Debt chapter, while for stocks in the Risk \& Return: Portfolio chapter.

## 2. BOND TERMINOLOGY \& VALUATION

### 2.1 Terminology

! Bond: A promised stream of CFs. The promissory agreement is traded on an exchange.
" Borrower/issuer: promises to pay future CFs for current CF
" Lender/buyer: Receives the promised CFs for current CF.
? So when two parties exchange ownership of a bond, what are they implicitly doing?
$\Rightarrow \quad$ An exchange of current CF for future CFs.

## Information provided at time of issue:

! Coupon: Equal periodic \$ CFs over life of bond (annual terms).
! Par Value: CF to be received at maturity, typically $\mathbf{\$ 1 , 0 0 0}$
! YTM: Yield To Maturity = required return on debt
R It is interest rate on the bond that is implied by the market price of the bond. It represents what the bond market thinks is a "fair" return if you were to hold the bond till maturity. Thus, one would, expect this rate to change as the market interest rates change.

R The factors determining this yield will be discussed in the chapter on Risk \& Return: Debt.
$!\quad \mathrm{k}_{\mathrm{d}}$ : The interest rate on a new bond.
Note. The book uses $\mathrm{k}_{\mathrm{d}}$ and YTM interchangeably
! Coupon Rate $(\%)=\frac{\text { coupon }}{\text { Par Value }} ;$ fixed at issue time $\Rightarrow$ coupon $=$ coupon rate $\times$ Par Value

Note. This rate is fixed at the time of issue.
$!$ Current Yield $=\frac{\text { coupon }}{\text { closing price }}$
! At par: When a bond is selling at price $=\operatorname{Par}$ Value $=\$ 1,000 \Leftrightarrow$ This would happen when the coupon rate $=\mathrm{YTM}$
! Discount Bond: When a bond is selling at price < Face Value $\leftrightarrow$ Coupon Rate < YTM.
! Premium Bond: When a bond is selling at price > Face Value $\leftrightarrow$ coupon rate > YTM .
! $\quad$ Par Value $=$ Face Value $=\$ 1,000$
! We will consider only level bonds--simple case; coupons are fixed by the coupon rate.

## DEBT MARKET

## PRIMARY MARKET



SECONDARY MARKET: Lender 1 sells bond to buyer 2


### 2.2 PRICING OF BONDS

\# Motivation: Given the CFs and the YTM on a bond, what should its price be?
\# Two types of CFs:


$$
\begin{array}{ll}
\text { Annuity }(\text { INT }) & =\text { coupons } \\
\text { single } & =\text { par value paid at maturity }=\$ 1,000
\end{array}
$$

| periods | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{2}$ | $\ldots$ | At <br> Maturity |
| :--- | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |
| CFs | 0 | coupon | coupon | $\ldots$ | coupon |
|  |  |  |  |  | + <br> face value |

! PV of Par Value
$=$ Par Value $\times$ PVIF $_{k_{d}, n}$
$=M \times\left[\frac{1}{\left(1+k_{d}\right)^{n}}\right]$
! PV of Coupons Annuity
$=$ Coupon $\times$ PVIFA $_{k_{d} n}$
$=I N T \times\left[\frac{1}{\left(1+k_{d}\right)}+\frac{1}{\left(1+k_{d}\right)^{2}}+\ldots+\frac{1}{\left(1+k_{d}\right)^{N}}\right]$
where, $\mathrm{M}=$ par value, $\mathrm{INT}=$ coupon

$$
\begin{aligned}
V_{B} & =\text { Theoretical Bond Value } \\
& =P V \text { of Coupon Annuity }+P V \text { of Par Value } \\
& =\operatorname{INT}\left[P V I F A_{k_{d} N}\right]+M\left[P V I F_{k_{d} N}\right]
\end{aligned}
$$

## Example: Calculating Price of a Bond

Given: A firm borrows $\$ 1,000$ by issuing a bond with coupon rate of $10 \%$, and promises to pay back the principal in 20 years. If the current market interest rates on a similar bond is $10 \%$, what is the value of this bond?

## Solution:

! PV of Par Value
$=1,000 \times \frac{1}{\left(1+k_{d}\right)^{20}}=\frac{1,000}{(1.10)^{20}}$
$=1,000 \times .14864=\underline{148.64}$
! PV of Coupons

$$
\begin{aligned}
& =(\$ 1,000 \times 10 \%) \times \text { PVIFA }_{k_{d d} N}=100 \times P V I F A_{10,20} \\
& =100 \times 8.5136=\underline{851.36}
\end{aligned}
$$

! Total Value of Bond $=148.64+851.36=\underline{\underline{\$ 1,000}}=\underline{\underline{\text { Par }}}$

Note: Since the bond is currently valued at par, if market interest rates $\left(\mathrm{k}_{\mathrm{d}}\right)$ increase, the bond would sell at a discount.
? Was there an easier way, in this example, to determine the price?
? Two bonds with identical coupons, would they necessarily have the same price?

### 2.3 BONDS WITH SEMIANNUAL COMPOUNDING

$\mathrm{k}_{\mathrm{d}}=10 \%$; matures in 10 years; $10 \%$ coupon, paid semiannually.
Solution:
Step 1: Determine CFs
$!$ coupon $=\frac{\text { annual coupon }}{\# \text { of coupon periods }}=\frac{\text { coupon rate } \times \text { face value }}{2}$

$$
=\frac{.1 \times 1,000}{2}=\frac{100}{2}
$$

! Face value $=1,000$

Step 2: Determine number of periods
! $\quad(10$ years to maturity) $\times(\#$ of periods in a year) $=10 \times 2=20$
Step 3: Use PV to find price of bond.

Value of Bond $=V_{B}$

$$
\begin{aligned}
& =\frac{\text { coupon }}{2}\left[\text { PVIFA }_{\frac{k_{d}}{2}, 2 N}\right]+1,000\left[P^{V_{I}} F_{\frac{k_{d}}{2}, 2 N}\right] \\
& =\frac{100}{2}\left[P V I F A_{\frac{10}{2}, 20}\right]+1,000\left[P V I F_{\frac{10}{2}, 20}\right]
\end{aligned}
$$

### 2.4 CALCULATING YTM ON A BOND

## Motivation:

What we want to do here is look at how you might come up with the interest rate (YTM) on any bond that is trading in the secondary market. Thus, we are reversing the question here; we are asking the following: Given the market price of a bond and its coupons, what is its implied YTM?

Remember the IRS \& ATT examples in the previous chapter! Well, the $4 \%$ and $6 \%$ used represent the YTM on a bond issued by the U.S. treasury that has 1 year to maturity, and a bond issued by ATT with 1-year to maturity, respectively.

## Example

Given: The following is information on a bond that is trading in the secondary market with the following characteristics:
coupon rate $=6 \%$; years to maturity $=30$; current market bond price $=\$ 1,153.72$
$\mathbf{Y T M}=$ ?

## Solution:

Step 1: calculate coupon

$$
\text { coupon }=\text { coupon rate } \times \text { par value }=6 \% \times \$ 1,000=\underline{\$ 60}
$$

Step 2: calculate YTM

$$
\begin{aligned}
P V & =\text { SUM of discounted CFs } \\
\Rightarrow \$ 1,153.75 & =60\left(P V I F A_{Y T M, 30}\right)+\$ 1,000\left(P V I F_{Y T M, 30}\right)
\end{aligned}
$$

This equation has two unknowns: PVIFA and PVIF. Thus, unfortunately, other than using a calculator or a PC, there isn't an easy formula to calculate YTM.

One solution is to use the Trial \& Error Method, which states: choose YTM such that

Left Hand Side = Right Hand Side of equation

Step 2A: Try YTM = 6\%
$\Rightarrow 60\left(\right.$ PVIFA $\left._{6,30}\right)+1,000\left(P V I F_{6,30}\right)$
$=60(13.7648)+1,000(.1741)=825.88+174.1=999.98$
but 999.98 < price $=\$ 1,153.72 \Rightarrow 6 \%$ is bad
Step 2B: You have to try a number $<6 \%$, say YTM $=\underline{4 \%}$

$$
\Rightarrow 60(17.2920)+1,000(.3083)=1,346.35>\text { price }
$$

If you try $\mathrm{YTM}=5 \%$, you will get it right.

$$
\mathrm{YTM}=\mathrm{k}_{\mathrm{d}}=\underline{\underline{5 \%}}
$$

Thus, at $\mathrm{YTM}=5 \%$,
$60 \times\left[\mathrm{PVIFA}_{5 \%, 30}\right]+1,000 \times\left[\mathrm{PVIF}_{5 \%, 30}\right]=1,153.72=$ price of bond

### 2.5 Bond Price Quotations

| CORPORATION BONDS Volume \$32,040,000 |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Bonds | Cur |  |  | Net |
|  | Yld | Vol | Close | Chg. |
| ... | ... | ... | ... | .. |
| AmMed 113/499 | 12.4 | 25 | 933/4 | $+1 / 4$ |
| ATT3f s90 | 4.0 | 10 | $97^{1 / 4}$ | +1/16 |
| ATT½97 | 6.7 | 1 | 821/8 | $-1 / 4$ |

Prices of September 29, 1989.
? $\quad\left(\right.$ Price $\left._{\text {Sept. 29 }}\right)>=<\left(\right.$ Price $\left._{\text {Sept. } 28}\right)$
$\boldsymbol{?}^{1} \quad\left(\mathrm{YTM}_{\text {Sept. 29 }}\right)>=<\left(\mathrm{YTM}_{\text {Sept. } 28}\right)$
! Issued by ATT
! $\quad 3 \frac{7}{8}$ is coupon rate

$$
\begin{aligned}
\Rightarrow \text { coupon }=I N T & =\left(3 \frac{7}{8} \%\right)(\$ 1,000) \\
& =(3.87 \%)(1,000)=(.03875)(1,000) \\
& =\$ 38.75
\end{aligned}
$$

- 90 = year of maturity
! $\quad$ Price at close $\left(97 \frac{1}{4}\right)$ is measured as $\%$ of par

$$
=\left(97 \frac{1}{4}\right) \times 1,000=\underline{972.5}=\text { market price at close }
$$

Thus, you need to pay $\$ 972.5$ to buy this bond
! Current yield $=38.75 / 972.5=\underline{4 \%}$
! Vol. is \# of bonds that changed hands.
Note. If you buy one bond, $\mathrm{Vol}=1$, since it is assumed that someone sold it.
! Net change:
Difference between today's closing price and the previous day's.
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### 2.6 V $_{\text {B }}$ AND THE 'MARKET PRICE.'

\# $\quad \mathbf{V}_{\mathbf{B}} \quad$ price of bond sum of discounted CFs.

> "theoretical" price "formula" price "should be " price "intrinsic" value
\# Market Mechanism: competition
If market price $>\mathrm{V}_{\mathrm{B}} \Rightarrow$ bond is overvalued $\Rightarrow$ investors sell the bond $\Rightarrow$ Supply for bond $\quad \Rightarrow$ market price until market price $=V_{B}$.
\# Obviously, if EMH (Efficient Market Hypothesis) holds, then there can be no such difference.
\# Some Street Jargon: [not responsible for it in exam]
! overvalued $\Leftrightarrow$ dear
! undervalued $\Leftrightarrow$ cheap

## 3. COMMON STOCK VALUATION

### 3.1 TWO WAYS OF LOOKING AT CFs:

! (A) Assume firm will be in business for a long time.
What are the relevant CFs when valuing equity?
$\mathrm{CF}=$ Dividends; not earnings since the latter does not necessarily go as cash to shareholders, if it is partially retained.

$$
\begin{align*}
P_{0} & =\text { sum of discounted cash flows } \\
& =\frac{\text { Dividend }_{1}}{\left(1+k_{s}\right)^{1}}+\frac{\text { Dividend }_{2}}{\left(1+k_{s}\right)^{2}}+\frac{\text { Dividend }_{3}}{\left(1+k_{s}\right)^{3}} \tag{1}
\end{align*}
$$

where,
Dividend ${ }_{t}$ is dividend per share paid at time $t$.
Note. Dividend $_{t}$ is expected future dividend in period "t." Thus, they need to be estimated/forecasted.
$\mathrm{k}_{\mathrm{s}}=$ required rate of return on equity
Thus,
Price of stock reflects all future market-expected CFs, i.e., price is forward looking. Thus, if a company's earnings are, for example, expected to $\quad \Rightarrow \mathrm{CF} \quad \Rightarrow$ price

## ! (B) Based on holding-period valuation:

From an investor's side; you buy a stock today at $\mathrm{P}_{0}$, then you sell it one year from today, at $P_{1}$ and you receive Dividend ${ }_{1}$ during the holding period. Thus, you receive two CFs: $\mathrm{P}_{1}$ and Dividend ${ }_{1}$.

$$
\begin{equation*}
\Rightarrow P_{0}=\frac{\text { Dividend }_{1}}{1+k_{s}}+\frac{P_{1}}{1+k_{s}} \tag{2}
\end{equation*}
$$

and

$$
P_{1}=\frac{\text { Dividend }_{2}}{\left(1+k_{s}\right)}+\frac{P_{2}}{\left(1+k_{s}\right)}
$$

## Show that (1) and (2) yield the same valuation:

Substituting for $\mathrm{P}_{1}$ in equation (2) gives

$$
P_{0}=\frac{\text { Dividend }_{1}}{1+k_{s}}+\frac{\text { Dividend }_{2}}{\left(1+k_{s}\right)^{2}}+\frac{P_{2}}{\left(1+k_{s}\right)^{2}}
$$

Continuing with the substitution for $\mathrm{P}_{2}$, then $\mathrm{P}_{3}$, etc., you will get the basic definition equation (1).
(1) and (2) yield equivalent values.

### 3.2 TWO SPECIAL CASES:

Limitations of above formulas: Hard, if not impossible, to estimate all future CFs (Dividends). Thus, to make the formulas useful, we need simplifying assumptions about expected dividend values.
! Case 1: Assume all Dividends are the equal, and paid over a long time horizon, then

$$
\begin{equation*}
P_{0}=\frac{C F}{k}=\frac{\text { Dividend }}{k_{s}} \tag{3}
\end{equation*}
$$

where
$\mathrm{k}_{\mathrm{s}}$ is the required rate of return.

## Definition: A stream of constant CFs paid indefinitely is called a perpetuity, which is a perpetual annuity.

## Example: Calculating Price of A Stock (perpetuity)

Given: Dividend $_{1}=$ Dividend $_{2}=\ldots=$ Dividend $_{n}=\$ 2, k_{s}=20 \%$

$$
\Rightarrow P_{0}=\frac{2}{.2}=\underline{\underline{\$ 10}}
$$

Alternatively: say you hold stock for two years, then you sell it

$$
P_{0}=\frac{\text { Dividend }_{1}}{1+k_{s}}+\frac{\text { Dividend }_{2}}{\left(1+k_{s}\right)^{2}}+\frac{\frac{\text { Dividend }}{k_{s}}}{\left(1+k_{s}\right)^{2}}=\underline{\underline{\$ 10}}
$$

! Case 2: Assume Dividend grows at a constant rate g, over a long time horizon, then

$$
\begin{equation*}
P_{0}=\frac{C F_{1}}{k-g}=\frac{\text { Dividend }_{1}}{k_{s}-g} \tag{4}
\end{equation*}
$$

## Practical considerations:

! Cannot use formula if $\mathrm{g}>$ or $=\mathrm{k}$, as price becomes meaningless.
! You can make other assumptions about how "g" might change over time and come up with other formulas:
R Case 3: Dividend first increasing then becomes constant
? Can you think of an interesting scenario?
R Case 4: Dividend increases but then slows down.
! Equations (3) and (4) and formulas using cases 3 and 4 are formulas in the sense that there is no intuition behind the result. It is purely a mathematical result. Thus, do not memorize them!
! Estimating "g": There are two approaches
R Based on analysts' forecasts of future growth rates.
A firm that provides such a service is Lynch, Jones \& Ryan in their Institutional Brokers Estimate System (IVB\E\S), which is a "consensus" forecast, i.e. average of analysts' forecasts. Another provider is Value Line.

R Using historical rates

| Year | Dividend | Change | \% change |
| ---: | :---: | :---: | :---: |
| 1987 | 1.10 | NA | NA |
| 88 | 1.20 | 0.10 | ${ }^{2} 9.09$ |
| 89 | 1.35 | 0.15 | 12.50 |
| 90 | 1.40 | 0.05 | 3.70 |
| 91 | 1.55 | 0.15 | 10.71 |

$$
\begin{aligned}
\hat{g} & =\text { estimate of } g=\text { average } \% \text { change } \\
& =\frac{9.09+12.5+3.7+10.71}{4}=\underline{\underline{9 \%}}
\end{aligned}
$$

How much would you pay for a share of a company that promises not to ever pay dividends?

## Example: Calculating Price of A Stock ( g is constant)

Given: Dividend $_{0}=\$ 2, g=10 \%, k_{s}=20 \%$

$$
\begin{aligned}
\Rightarrow P_{0} & =\frac{\text { Dividend }_{1}}{k_{s}-g} \\
& =\frac{\text { Dividend }_{0} \times(1+g)}{k_{s}-g} \\
& =\frac{\$ 2 \times(1+.1)}{.2-.1}=\frac{2.2}{.1}=\underline{\underline{\$ 22}}
\end{aligned}
$$

## Example: Calculating Future Prices of A Perpetuity Stock

Given: $\quad k_{s}=10 \%$, Dividend $=\$ 1, \mathrm{~g}=0$

$$
\begin{aligned}
p_{0}= & \frac{\text { Dividend }}{k_{s}}=\frac{\$ 1}{.1}=\underline{\underline{\$ 10}} \\
p_{1}= & \frac{\text { Dividend }}{k_{s}}=\frac{\$ 1}{.1}=\underline{\underline{\$ 10}} \\
& \Rightarrow p \text { does not change }
\end{aligned}
$$

? So why would anyone buy this stock?
There is an additional CF; dividends. Thus,

$$
\begin{aligned}
\text { :tual return } & =\frac{\left(p_{1}-p_{0}\right)+\text { Dividend }}{p_{0}}=\frac{0+1}{10}=10 \\
& =k_{s}
\end{aligned}
$$

## Example: price of a stock if $\mathbf{k}_{s}$ Increases

Suppose now $\mathrm{k}_{\mathrm{s}}$ increases after 1-year to $20 \%$

$$
p_{1}=\frac{\text { Dividend }}{k_{s}}=\frac{1}{.2}=\underline{\underline{\$ 5}}
$$

capital gains $=$ profit $=5-10=-\$ 5$
You lose money.

### 3.3 ESTIMATING REQUIRED RETURN ON A STOCK

! Capital Asset pricing Model (CAPM) approach later in the course
! Using Discounted CF (DCF ):from equation (4) above, we have:

$$
\begin{aligned}
p_{0} & =\frac{\text { Dividend }_{1}}{k_{s}-g} \\
\Rightarrow & \left(k_{s}-g\right) \times P_{0}=\text { Dividend }_{1} \\
& \Rightarrow k_{s} P_{0}-g P_{0}=\text { Dividend }_{1} \\
\Rightarrow & k_{s}=\frac{\text { Dividend }_{1}+g P_{0}}{P_{0}} \\
\Rightarrow k_{s} & =\frac{\text { Dividend }_{1}}{P_{0}}+g
\end{aligned}
$$

$$
k_{s}=\text { dividend yield }+ \text { expected growth rate }
$$

## Bond \& Stock Valuation

## Example: Calculating $\mathrm{k}_{s}$ using DCF

Given: Dividend $_{0}=\$ 4.19, \mathrm{~g}=5 \%, \mathrm{p}_{0}=\$ 50$

## Solution:

$$
\begin{aligned}
k_{s} & =\frac{\text { Dividend }_{0} \times(1+g)}{p_{0}}+g \\
& =\frac{\$ 4.19 \times(1.05)}{50}+0.05 \\
& =0.088+0.05=\underline{\underline{13.8 \%}}
\end{aligned}
$$

### 3.4 WSJ EQUITY REPORTING

52 Weeks


Net
© T/F: 11-14; problems: 4-7©

## 4. PREFERRED STOCK

### 4.1 TERMINOLOGY

! Has precedence over common stock in the payment of dividends in case of liquidation.
! Stated Value: Value to be paid to holders in event of liquidation
! Cumulative Dividends: Firm's directors can vote to omit the dividends. However, They accumulate.

### 4.2 WHY ISSUE PREFERRED STOCK?

\# Advantage to common holders:
No voting right for preferred, i.e. no loss of control; solution to equity dilution problem
\# Advantage to buyers:
! Only $30 \%$ of dividend income is taxable for a corporation.
! Insurance companies have additional regulatory incentives to hold them.

### 4.3 Valuation

Since the CFs of a preferred stock constitute a perpetuity, then

$$
p_{p s}=\frac{\text { Dividend }_{p s}}{k_{p s}}
$$

## 5. EMH IMPLICATIONS ON INVESTMENTS

## ! Equity Market Implications

All stocks are fairly priced in the market $\Rightarrow$ An investor chooses stocks based on her appetite for risk

$$
\Leftrightarrow
$$

There are neither "bargain" nor "expensive" stocks
$\Rightarrow$ An average risk portfolio can be attained by randomly throwing about 500 darts at the WSJ Buying an index mutual fund from Vanguard, or Schwab, or Fidelity, or ...[When we talk about mutual funds, later in the semester, you will appreciate why the second proposition makes more sense!]
? Do mutual funds, in general, believe in the EMH?
! Bond Market Implications
Individual chooses bond(s) depending on her tolerance for risk.

## 6. SUMMARY

$\sqrt{\text { Value of bond }=\text { SUM of discounted CFs }}$

$$
=\mathrm{PV} \text { of coupons }+\mathrm{PV} \text { of Par Value }
$$

Two Applications:
Given the market price on a bond, what is its YTM?
Given the YTM, what is should its price be?
$\sqrt{ }$ Vocabulary: coupon, coupon rate, $\mathrm{k}_{\mathrm{d}}$, discount, premium, par, dividend yield, YTM.
$P_{0}=$ Sum of Discounted Cash Flows
$=\frac{\text { Dividend }}{k_{s}}$, when all future dividends are assumed equal
$=\frac{\text { Dividend } d_{1}}{k_{s}-g}$, when Dividend is assumed to grow at rate $g$

## $\checkmark$ Dynamic Mechanism: competition

If market price $>\mathrm{V}_{\mathrm{B}} \Rightarrow$ bond is overvalued $\Rightarrow$ investors sell the bond $\Rightarrow$ supply of $\Rightarrow$ market price until market price $=\mathrm{V}_{\mathrm{B}}$.

Thus, for a typical investor, EMH holds.
$\sqrt{ } \quad \mathrm{k}_{\mathrm{s}}$ can be estimated from DCF, as above, or CAPM method discussed later in the course.

EMH and investment implications

## 7. IN FUTURE CLASSES

O Calculating bond prices with different features: call, put, sinking fund.
O More on portfolio management
O Transactions costs

## 8. ADDITIONAL READINGS

Smith and Williams, "Experimental Market Economics," Scientific America, December 1992.

Malkiel. A Random Walk Down Wall Street. Norton, 1990.

## 9. QUESTIONS

## I. Agree/Disagree Explain

1. If the market price of a bond is greater than the theoretical (formula) price, the bond is overvalued.
2. If $k_{d}$ increases, then the price of a bond has to decrease.
3. If $\mathrm{k}_{\mathrm{d}}(\mathrm{YTM})$ increases, then holders of bonds are worse off while those who had issued them are better off.
4. Knucklehead: "Buy government bonds. You cannot lose money. They are guaranteed by the government."
5. The coupon rate changes as interest rates change.
6. If interest rates remain constant, bond prices remain constant over the life of the bond, other things equal.
7. Regular payments on a loan constitute contributions to both principal and interest, while those on a simple bond, issued at par, are contributions to interest only.
8. If $\mathrm{k}_{\mathrm{d}}$, then a bond holder will necessarily realize a profit.
9. If a bond is undervalued, it suggests that it is not valuable enough to buy.
10. If a bond is undervalued, then it is selling at a discount.
11. Two investors are evaluating IBM's stock for possible purchase. They agree on the expected value of Dividend ${ }_{1}$ and also on the expected future dividend growth rate. Further, they agree on the riskiness of the stock. However, one investor normally holds stocks for 2 years, while the other holds stocks for 10 years. Thus, they should both be willing to pay the same price for IBM's stock.
12. For a given level of dividends, the higher the riskiness of dividends, the higher the market price to compensate for risk.
13. It does not make sense to invest in a stock whose price is not appreciating.
14. It does not make sense to invest in a stock that has no growth in earnings.

## II. NUMERICAL

1. Given a bond with coupon rate $=10 \%$, paid semiannually, $\mathrm{YTM}=16 \%$, and matures in 3 years. What is the price of the bond? What is the effective annual yield?
2. Given bonds A, B, and C that matures in 2 years and has no coupon payments in the mean time. The bond is currently selling at $\$ 500$ and has a par value of $\$ 1,000$. What is the YTM on the bond?
3. Given: A bond with $10 \%$ coupon rate, and the following yield curve: (use 6bonds.wk3)

| Bond | $\underline{\text { Maturity (yrs) }}$ | $\underline{Y T M}$ |
| :--- | :--- | :--- | :--- |
| A | 1 | $8 \%$ |
| B | 2 | 10 |
| C | 3 | 12 |

(a) Can all these bonds be selling at par?
(b) Calculate the prices of these bonds.
(c) Suppose that the yield curve changes, such that the yields on the 3-year bond decreases by $1 \%$, the yield on the 2 -year bond remains the same, and that for the 1 -year bond increases by $1 \%$. What are the new bond prices?
(d) Which is more sensitive to interest rates, give data in (c)? Why? (Note. Sensitivity can be measured as: " \% change in price for a $1 \%$ change in interest rates", i.e.

$$
\text { sensitivity }=\frac{\text { change in price }}{\text { original price }}=\frac{p_{1}-p_{0}}{p_{0}}
$$

(e) Would the bond holders make money with the new yields?
(f) If the market prices of these bonds were $\$ 900, \$ 1,005$, and $\$ 990$ respectively, are they under or over-valued?
(g) If the yield curve stays at the original level in (a) over the next year, what would be their prices one year from today?
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4. If $\mathrm{g}=0$, and Dividend ${ }_{1}=1$, required rate of return $=10 \%$, and the market price is $\$ 12$. Is this stock over or under valued?
5. You buy a share of Ciao corporation stock for $\$ 21.40$. You expect it to pay dividends of $\$ 1.07$, and $\$ 1.1449$ in Years 1, and 2 respectively.
a. Calculate the growth rate in dividends.
b. Calculate the expected dividend yield.
c. Assuming that the calculated growth rate is expected to continue. What is this stock's expected total rate of return?
6. Given the following information on stock XYZ Inc.: Dividend $_{0}=.1, \mathrm{~g}=0, \mathrm{k}_{\mathrm{s}}=10 \%$. Calculate the current price and the expected price of the stock one year from today.
7. Given the following information on stock XYZ Inc.: Dividend $_{1}=.1, \mathrm{~g}=5 \%, \mathrm{k}_{\mathrm{s}}=10 \%$.
(a) Calculate $\mathrm{p}_{0}, \mathrm{p}_{1}$, and $\mathrm{p}_{2}$.
(b) What is the growth rate in price?

## ANSWERS TO QUESTIONS

## I. Agree/Disagree Explain

1. Agree. If you were to buy this bond (asset), then you would pay more than it is worth.
2. Agree. From definition of price of bond. Price and $\mathrm{k}_{\mathrm{d}}$ are inversely related.
3. Agree. Two ways of looking at it.
(a) As YTM increases, then the price of the bond decreases. Thus, as a holder (buyer/lender) you would lose money if you were to sell the bond after the increase in YTM. Thus, you would be worse off.
(b) If YTM increases, the holder would now get lower returns than comparable new bonds.
(c) From sellers side: For the same amount to be borrowed, they have to pay higher interest (cost). Thus, they would be better off.
4. Disagree. Your CFs--amount and timing, but not their risk-- are guaranteed, not the price. Obviously, as YTM changes, so would the price on the bond.
5. Disagree. Coupon rate is fixed. Interest rate (YTM) changes.
6. Disagree. If the bond is selling at a discount, then its price goes up to par $(\$ 1,000)$ on maturity day. On the other hand, if premium, its price does down to $\$ 1,000$. Only if it is at par would its price stay the same.
7. Agree. This is one of the differences between a simple loan and a simple bond. Another is that the latter is traditionally trade on an exchange.
8. Disagree. You do not know the price the investor had to pay for it $\left(\mathrm{p}_{0}\right)$. Remember profit is $\left(\mathrm{p}_{1}-\mathrm{p}_{0}\right)$. Obviously the investor is better off after the decrease in $\mathrm{k}_{\mathrm{d}}$. Thus, if they were incurring a loss, new the loss is less.
9. Disagree. Actually such a bond would be a great buy, as its market price < theoretical price.
10. Disagree. Discount or premium have nothing to do with under or over-valuation. The motivation is to emphasize that under-/over-valued has to do with the relationship between market price and "theoretical" price, while discount/premium have nothing to do with the quality of the bond. They indicate a bond's price relative to par=\$1,000.
11. Agree. Since both agree on the CFs and the k. Both should come up with the same PV.
12. Disagree. The higher the risk, the higher the required return. Thus, since price is equal to "sum of discounted CFs", with the same amount of dividend, but riskier, we get a lower price. Note once again risk and price are inversely related.
13. Disagree. There are two sources of return: capital gains and dividends. As long as you receive dividends, you are realizing dividend yield, thus positive returns.
14. Disagree. Growth in earnings does not mean that the price is increasing. With no growth, you receive a constant stream of dividends, thus positive returns.

## II. Problems

1. Given a bond with coupon rate $=10 \%$, paid semiannually, $\mathrm{YTM}=16 \%$, and matures in 3 years. What is the price of the bond?

$$
\text { coupon }=\frac{1 \mathrm{U} \% \times 1, \mathrm{UUU}}{2}=\$ 50
$$

$$
\text { semiannual interest rate }=\frac{16 \%}{2}=8 \%
$$

number of periods $=3$ years $\times 2$ periods in a year $=6$

$$
\Rightarrow p_{B}=50\left(P V I F A_{\frac{16 \%}{2}, 6}\right)+1,000\left(\text { PVIF }_{\frac{16 \%}{2}, 6}\right)
$$

$$
\begin{aligned}
E A R & =\left(1+\frac{A P R}{m}\right)^{m}-1 \\
& =\left(1+\frac{A P R}{2}\right)^{2}-1=\underline{\underline{16.4 \%}}
\end{aligned}
$$

2. Given a bond that matures in 2 years and has no coupon payments in the mean time. The bond is currently selling at $\$ 500$ and a par value of $\$ 1,000$. What is the YTM on the bond?

$$
\begin{gathered}
p=\frac{\text { par value }}{(1+k)^{2}} \Rightarrow \\
500=\frac{\$ 1,000}{(1+k)^{2}} \Rightarrow \\
(1+k)^{2}=\frac{1,000}{500}=2 \Rightarrow \\
(1+k)=\sqrt{2} \Rightarrow k=\sqrt{2}-1=1.4-1=.4=\underline{\underline{40 \%}}
\end{gathered}
$$

3. (a) No since the coupon rates and the YTM are not equal.
(b) $\$ 1,018.5, \$ 1,000$, and 951.96 respectively.

$$
\begin{aligned}
& \mathrm{P}_{\mathrm{A}}=(\$ 1,000)(\text { coupon rate })\left[\mathrm{PVIFA}_{\mathrm{YTM}, \mathrm{n}}\right]+\$ 1,000\left[\mathrm{PVIF}_{\mathrm{YTM}, \mathrm{n}}\right] \\
&=(\$ 1,000)(10 \%)\left[\mathrm{PVIFA}_{8 \%, 1}\right]+\$ 1,000\left[\mathrm{PVIF}_{8 \%, 1}\right] \\
&=\$ 1,018.5 \\
& \mathrm{P}_{\mathrm{B}} \quad=(\$ 1,000)(10 \%)\left[\mathrm{PVIFA}_{10 \%, 2}\right]+\$ 1,000\left[\mathrm{PVIF}_{10 \%, 2}\right] \\
&=\$ 1,000 \\
& \mathrm{P}_{\mathrm{C}} \quad=(\$ 1,000)(10 \%)\left[\mathrm{PVIFA}_{12 \%, 3}\right]+\$ 1,000\left[\mathrm{PVIF}_{12 \%, 3}\right] \\
&=\$ 951.96
\end{aligned}
$$

(c) $1,009.2, \$ 1,000$, and 975.6 respectively.
(d) Bond C is.

## Bond A

sensitivity $=\mathrm{P}_{1}-\mathrm{P}_{0} / \mathrm{P}_{0}$
1009.14-1018.49 / $1018.49=-.9 \%$

Bond C
$755.44-951.98 / 951.98=-21 \%$
(e) Bond A holders lose, bond B holders are unaffected, and C holders are better off.
(f) A is under-valued, its market value < theoretical value B is over-valued, its market value $>$ theoretical value

C is over-valued. its market value $>$ theoretical value
(g) A has matured, C would be par, and B would be at \$1,018.

$$
\begin{aligned}
\mathrm{P}_{\mathrm{B}} & =(\$ 1,000)(10 \%)\left[\mathrm{PVIFA}_{8 \%, 1}\right]+\$ 1,000\left[\mathrm{PVIF}_{8 \%, 1}\right] \\
\mathrm{P}_{\mathrm{C}} & =(\$ 1,000)(10 \%)\left[\mathrm{PVIFA}_{10 \%, 2}\right]+\$ 1,000\left[\mathrm{PVIF}_{10 \%, 2}\right] \\
& =\$ 1,000
\end{aligned}
$$

## CONCLUSIONS

1. A premium bond is when coupon rate $>$ YTM.
2. If YTM bond price
3. The longer the maturity, the higher the interest rate sensitivity (risk).
4. As the bond moves closer to maturity, assuming interest rates and default risk are constant, then
i) If the bond starts at a premium, then P .
ii) if the bond starts at par, then P .
iii) if the bond starts at a discount, then P .
5. If $g=0$, and $D_{1}=1$, required rate of return $=10 \%$, and the market price is $\$ 12$. Is this stock over or under valued?

Answer.
Step 1: Compare theoretical (required) value to market value.

Step 2: Choose the correct formula to calculate theoretical value. Since beta is not given, then CAPM cannot be used. Thus, use the formula based on discounted CF with a constant dividend growth rate $g$.

$$
\begin{gathered}
\text { theoretical price }=\frac{D_{1}}{k_{s}-g}=\frac{1}{.1-0}=10<\text { market price } \\
\Rightarrow \underline{\text { over-valued }}
\end{gathered}
$$

Thus, it is over-valued. Investors would sell it (demand goes down), thus bidding down the price till the theoretical (formula) price $=$ market price .
5. You buy a share of Ciao corporation stock for $\$ 21.40$. You expect it to pay dividends of $\$ 1.07$, and $\$ 1.1449$ in Years 1 , and 2 respectively.
a. Calculate the growth rate in dividends.
b. Calculate the expected dividend yield.
c. Assuming that the calculated growth rate is expected to continue. What is this stock's expected total rate of return?
(a) Two possible approaches:
\# Approach 1: From definition of growth rate;

$$
\begin{aligned}
\text { growth rate }=g & =\frac{\text { Dividend }_{1}-\text { Dividend }_{0}}{\text { Dividend }_{0}} \\
& =\frac{\text { Dividend }_{1}}{\text { Dividend }_{0}}-\frac{\text { Dividend }_{0}}{\text { Dividend }_{0}} \\
& =\frac{\text { Dividend }_{1}}{\text { Dividend }_{0}}-1 \\
& =\frac{1.1449}{1.07}-1=\underline{\underline{7 \%}}
\end{aligned}
$$

\# Approach 2: Using FV;

$$
\begin{aligned}
& F V_{1}=C F \times(1+g) \\
& \quad \Rightarrow g=\frac{F V_{1}}{C F}-1=\frac{1.1449}{1.07}-1=\underline{\underline{7 \%}}
\end{aligned}
$$

(b) dividend yield $=\frac{D_{0}}{P_{0}}=\frac{1.07}{21.4}=\underline{\underline{5 \%}}$
(c) Need to calculate required return not realized return. Thus, use formula from DCF.

$$
k_{\text {Ciao }}=\text { dividend yield }+9=5+7=\underline{\underline{12 \%}}
$$

6. 

$$
\begin{gathered}
p_{0}=\frac{D_{0}}{k_{s}}=\frac{.1}{10 \%}=\underline{\underline{\$ 1}} \\
p_{1}=\frac{.1}{10 \%}=\$ 1
\end{gathered}
$$

7. a)

$$
\begin{aligned}
& p_{0}=\frac{D_{1}}{k_{s}-g}=\frac{.1}{10 \%-5 \%}=\underline{\underline{\$ 2}} \\
& p_{1}=\frac{D_{2}}{k_{s}-g}=\frac{D_{1}(1+g)}{10 \%-5 \%}=\frac{.1(1+.05)}{.05}=\underline{\underline{\$ 2.1}} \\
& p_{2}=\frac{D_{3}}{k_{s}-g}=\frac{D_{1}(1+g)^{2}}{10 \%-5 \%}=\frac{.1(1+.05)^{2}}{.05}=\$ 2.21
\end{aligned}
$$

b)

$$
\text { growth in price }=\frac{p_{1}-p_{0}}{p_{0}}=5 \%
$$

## Endnotes

1. Since the price change is "+," it means that the price increased from the previous day's close. But, price and YTM are inversely related. Thus, the YTM must have decreased.
2. \% change $=\left(\right.$ Dividend $_{1}-$ Dividend $\left._{0}\right) /$ Dividend $_{0}$ $=(1.20-1.10) / 1.10=9.09$
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Bond \& Stock Valuation

## ELIMINATIONS

## a.i ESTIMATING STOCK CF

\# Just apply the valuation equation; DCF
\# Dividend $=(1-\%$ of earnings retained $) \times$ EPS $=(1-R R) \times$ EPS $=$ (payout ratio)(EPS)
! Firms tend to have a stable retention ratio
! Estimate (forecast) g
\# k can be obtained from the CAPM
\# If you assume dividends are growing at a constant rate g ,

$$
\begin{aligned}
& p=\frac{\text { Dividend }}{k_{s}-g}=\frac{(1-R R) \times E P S}{k_{s}-\hat{g}} \\
& \text { where, } \\
& \qquad \begin{aligned}
\hat{g} & =\text { forecasted growth rate in EPS, } \\
R R & =\text { Retention Ratio }=\% \text { of Earnings retained }
\end{aligned}
\end{aligned}
$$

T/F explain:
BEAVIS: "Buy GNMA securities! You cannot lose. They are guaranteed by the government."

Disagree. Remember from (Ch 13), the government only guarantees the amount and the timing of the CFs. As mortgage rates change, so do the prices of the mortgage-backedsecurities.

Young employees should, other things equal, put all their $401(\mathrm{k})$ contributions in stocks, since they yield higher returns in the long run.
(a) It is Agree that stocks have higher returns (riskier) than bonds.
(b) There is no theory that tells you where to put your money. It depends on your own attitudes towards risk.
(c) Empirical evidence suggests that a (60/40) or a (70/30) ratio of equity to debt has performed better than just equity or just debt. Also, equity should be in a well diversified portfolio. If your company does not provide an indexed fund, then you are probably better off investing in a number of equity mutual funds to obtain diversification.

## III. Is the market reaction to the following Bearish or Bullish?

(a) Unexpected decrease in interest rates.
(b) Unexpected inflation increases.
(c) Increase in companies' earnings.
(d) U.S. trade deficit increases.
(e) U.S. budget deficit increases.
(f) Economic boom.

